Discrete Math

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Grades 9 - 12

Executive summary:

It is our intent to use the lessons in this unit to enrich our current curriculums, and not be used necessarily in any given order, but when when it comes up in our classroom. The activities we have chosen cover the following standards:

NCTM Standard 3: Mathematics as Reasoning.
In grades 9-12, the mathematics curriculum should include numerous and varied experiences that reinforce and extend logical reasoning skills so that all students can--

- make and test conjectures;
- formulate counterexamples;
- follow logical arguments;
- judge the validity of arguments;
- construct simple valid arguments; and so that, in addition, college-intending students can--
- construct proofs for mathematical assertions, including indirect proofs and proofs by mathematical induction.

NCTM Standard for Geometry Use visualization, spatial reasoning, and geometric modeling to solve problems use vertex-edge graphs to model and solve problems;
7.4.1.1 Design simple experiments and collect data. Determine mean, median and range for quantitative data and from data represented in a display. Use these quantities to draw conclusions about the data, compare different data sets, and make predictions.
9.3.1.3 Understand that quantities associated with physical measurements must be assigned units; apply such units correctly in expressions, equations and problem solutions that involve measurements; and convert between measurement systems.
9.3.1.5 Make reasonable estimates and judgments about the accuracy of values resulting from calculations involving measurements.
9.2.2.1 Represent and solve problems in various contexts using linear and quadratic functions.

Overview of Days

Harry Potter - Logic puzzle

Sudoku - Logic puzzle

Logic Grid Puzzle - Logic puzzle

Streets of Ivanhoe - Vertex-Edge Graphs are used to find a driving path throughout town

Snow Plow in Champlin - Vertex-Edge Graphs are used to plow snow

Racing Dreams - Students will use a race simulator to change variables and determine their effectiveness.

M & M Prediction Students will use what is in a small bag to predict what is in a larger bag

Greatest Yankee: Students will rank 5 baseball players using statistics

Creating a Beautiful Class - The students had to find area of a classroom to remodel it

Al Gebra's Nightclub - Quadratic equations to find how big of a room is needed to seat 1153

Students will work with numerous situations to practice, explore, and understand many different examples of discrete math. Students will have the opportunity to learn useful real world and problem solving strategies. The students will work as an individual, in small groups and as a class to discover the properties and relationships of numbers.

MCA Questions they should be able to answer after the unit:

25. The number of students of each age on a bus is shown in the table.

Ages of Students

Age (years)	Number of Students
13	2
14	10
15	5
16	18
17	24

What is the median age of the students?

A. 10 years
B. 14 years
C. 15 years
D. 16 years

24. Anya listed the prices of meals on a menu.

$14.85 $10.75 $8.50 $12.45 $9.20

What is the mean price of the meals?

A. $6.35
B. $8.50
C. $10.75
D. $11.15

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Lesson: Harry Potter

Benchmark: NCTM Standard 3: Mathematics as Reasoning.
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-]make and test conjectures;
  - formulate counterexamples;
  - follow logical arguments;
  - judge the validity of arguments;
  - construct simple valid arguments; and so that, in addition, college-intending students can--
  - construct proofs for mathematical assertions, including indirect proofs and proofs by mathematical induction.

Launch: In the book *Harry Potter and the Scorer's Stone* there is a scene that they need to pick a particular bottle by reading clues.
Harry Potter Worksheet

Explore: Students in groups of 2-3 will explore the given logic problem. Questions to ask: What are somethings you have to be careful of when solving this problem? If a student seems to get to an answer fairly quickly be sure that they can justify and explain why to another student. What strategy are you using?

Share: After teachers observe the students working, ask for volunteers to show the class how they arrived at their answer.

Discuss what strategies were effective and which weren't and why or why not?

Summarize: Today we looked at some different ways to solve a puzzle given clues, and decided what ways were effective, which weren't, and things to look out for. We also had to create arguments for or against the logic we were trying to use.

Lesson: Sudoku

Benchmark: NCTM Standard 3: Mathematics as Reasoning.
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-]make and test conjectures;
  - formulate counterexamples;
  - follow logical arguments;
  - judge the validity of arguments;
  - construct simple valid arguments; and so that, in addition, college-intending students can--
  - construct proofs for mathematical assertions, including indirect proofs and proofs by mathematical induction.

Launch: The classic Sudoku game involves a grid of 81 squares. The grid is divided into nine blocks, each containing nine squares. The rules of the game are simple: each of the nine blocks has to contain all the numbers 1-9 within its squares. Each number can only appear once in a row, column or box.
Sudoku Puzzles

Explore: Start this logic puzzle individually, then gradually move it up, but no more than a group of 4. As students are working questions to ask: Ask how they know a particular number has to go in a spot, or why a number can't be somewhere?

Share: Have a few groups show their answers see if there are any similarities or differences, any mistakes that others may see. Discuss the strategies that groups and and individuals used and the pros and cons of each method.

Summarize: Today we looked at a sudoku puzzle to explore other methods for solving a logic puzzle this one with numbers rather than words like the Harry Potter. We found that it's helpful to remember look at each individual box and both columns and rows of numbers. Starting with 1 and working up to 9, or trying to find a given box first are all good strategies.

Lesson: Logic Grid Puzzle

Benchmark: NCTM Standard 3: Mathematics as Reasoning.
In grades 9-12, the mathematics curriculum should include numerous and varied experiences that reinforce and extend logical reasoning skills so that all students can--

-]make and test conjectures;
- formulate counterexamples;
- follow logical arguments;
- judge the validity of arguments;
- construct simple valid arguments; and so that, in addition, college-intending students can--
- construct proofs for mathematical assertions, including indirect proofs and proofs by mathematical induction.

Launch: We have looked at both the Harry Potter logic, and a sudoku logic puzzle. We are now going to look at a logic puzzle that you need to match up certain items. 1 grid is attached that you may or may not use your choice.
Daddy Goes Shopping

Explore: Students in groups of 2-3 will work through the logic puzzle. As students are working some questions to ask: How do you know or not know someone or something can or can't be matched up?

Share: Have a few groups share their answers especially any that may have differing answers. What strategies worked well, or didn't work well? What strategies worked the best for you? Which process did you like the most and why? Which puzzle did you like the most and why?

Summarize: We have been working on logic puzzles. While doing them we used different strategies to solve them. While using these strategies we had to develop arguments for or against why it was logical.

Lesson: Streets of Ivanhoe

Benchmark: NCTM Standard for Geometry Use visualization, spatial reasoning, and geometric modeling to solve problems use vertex-edge graphs to model and solve problems;

Launch: Deputy Doug wants to cover every street in the little town of Ivanhoe each shift that he is working. Deputy Doug also has to answer to the City Council on his budget. He doesn't like it when they question him on how much gas he uses. So, Deputy Doug wants to cover every street in the most efficient way. How can Deputy Doug drive down every street and what is the least amount of streets that he will cover more than once?

Explore: The students will be given just the first sheet on the first day. They will work in groups of four to try different methods of solving this problem.

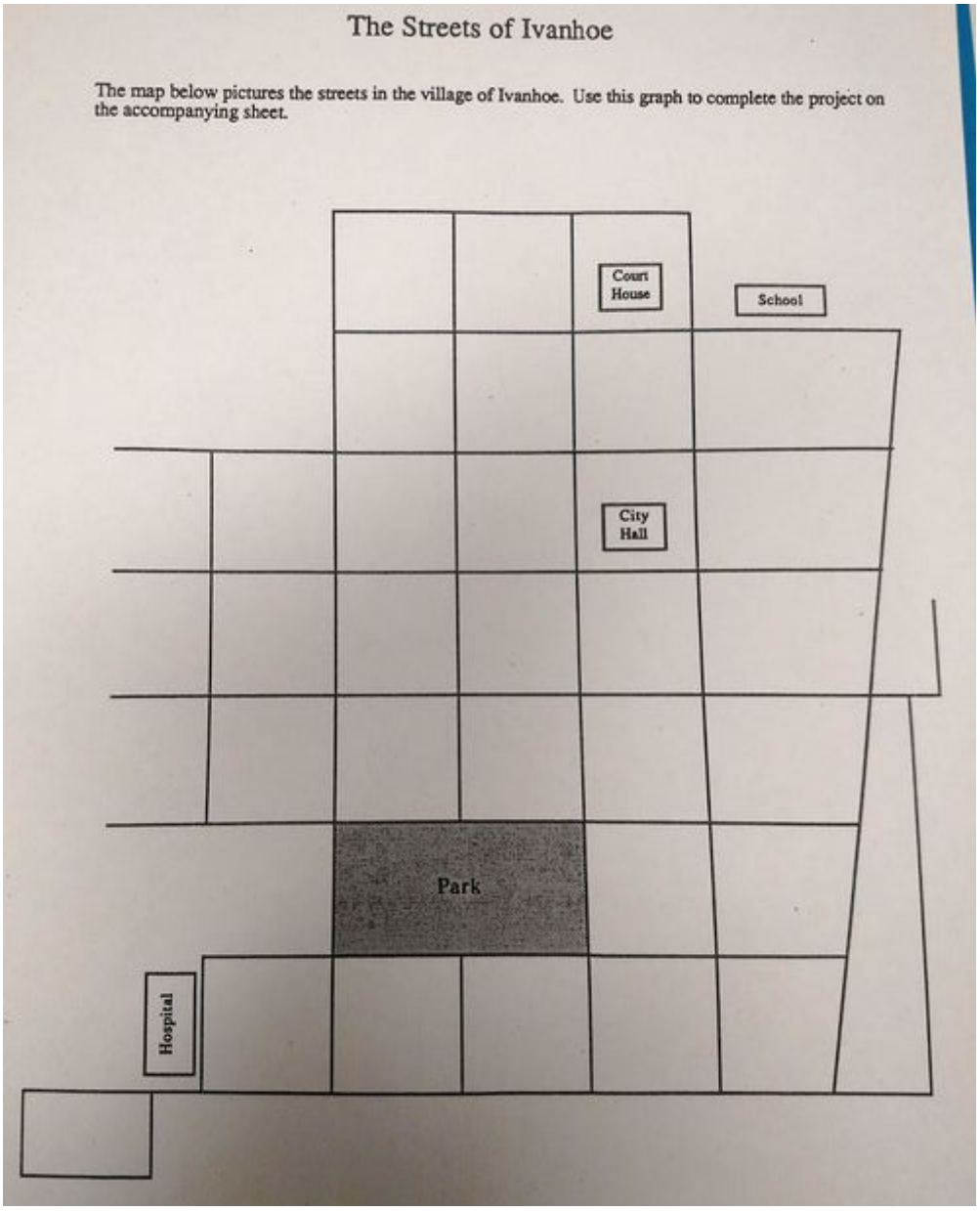
On the second day, the second sheet will be given to them. The students will continue to work in their groups with the new information.

Share: After making observations the teacher should select a few pieces of sample work. Ask those students to present how they arrived at their answers.

Summarize: We started with some basic vertex-edge graphs. We then switched to working with Deputy Doug and figuring out the best way for him to be able to drive on every street in Ivanhoe on each shift. We even were able to get the City Council to add some streets to the city in hopes to make this easier for Deputy Doug. What was the lowest number of streets that were driven on more than once? We found that we needed to have all the vertices with even amount of roads coming from them except for 2, the one we started on and the one we ended on.

The Streets of Ivanhoe

The map below pictures the streets in the village of Ivanhoe. Use this graph to complete the project on the accompanying sheet.

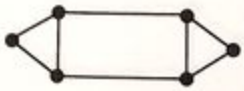


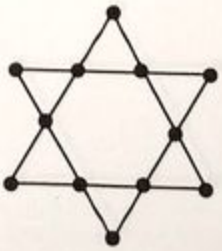
1. Barney Fife, the chief of police in Ivanhoe, wants to design a plan so he can patrol every street in the village of Ivanhoe as efficiently as possible. Therefore, he wants to be able to travel every street at least once, doubling back on as few streets as possible. Indicate on the "Streets of Ivanhoe" sheet what streets he should travel twice to efficiently patrol the village streets.

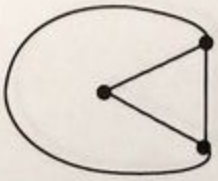
2. Barney was able to convince the taxpayers of the village to appropriate $150,000 for the addition of some new streets to make patrolling the village more efficient. The city engineer estimates that the cost is about $12,000 per block for a new street. The cost for a street shorter than a typical block goes down slightly, but not significantly. So if you add streets shorter than a typical block, figure the cost at $12,000 also. Streets longer than a typical block average out to about $20,000. On the "Streets of Ivanhoe" sheet indicate where you would add streets so as to make patrolling more efficient while still staying within your budget.

Sample Vertex-Edge Graphs

Name _____







Lesson: Snow Plowing the Streets of Champlin

Benchmark: NCTM Standard for Geometry Use visualization, spatial reasoning, and geometric modeling to solve problems, use vertex-edge graphs to model and solve problems;

Launch: We have worked on the Streets of Ivanhoe and found that we needed to have intersections that had most of the streets with an even amount of streets at each junction. Today we are going to be working to develop a plan to snow plow the Streets of Champlin. Hand out the sheets with directions and the map of the designated area. What is going to be the most efficient route (least amount of streets being plowed multiple times) to get all the streets cleared?

Explore: Students will work in pairs to explore solutions to the worksheet problem. Worksheet is following.

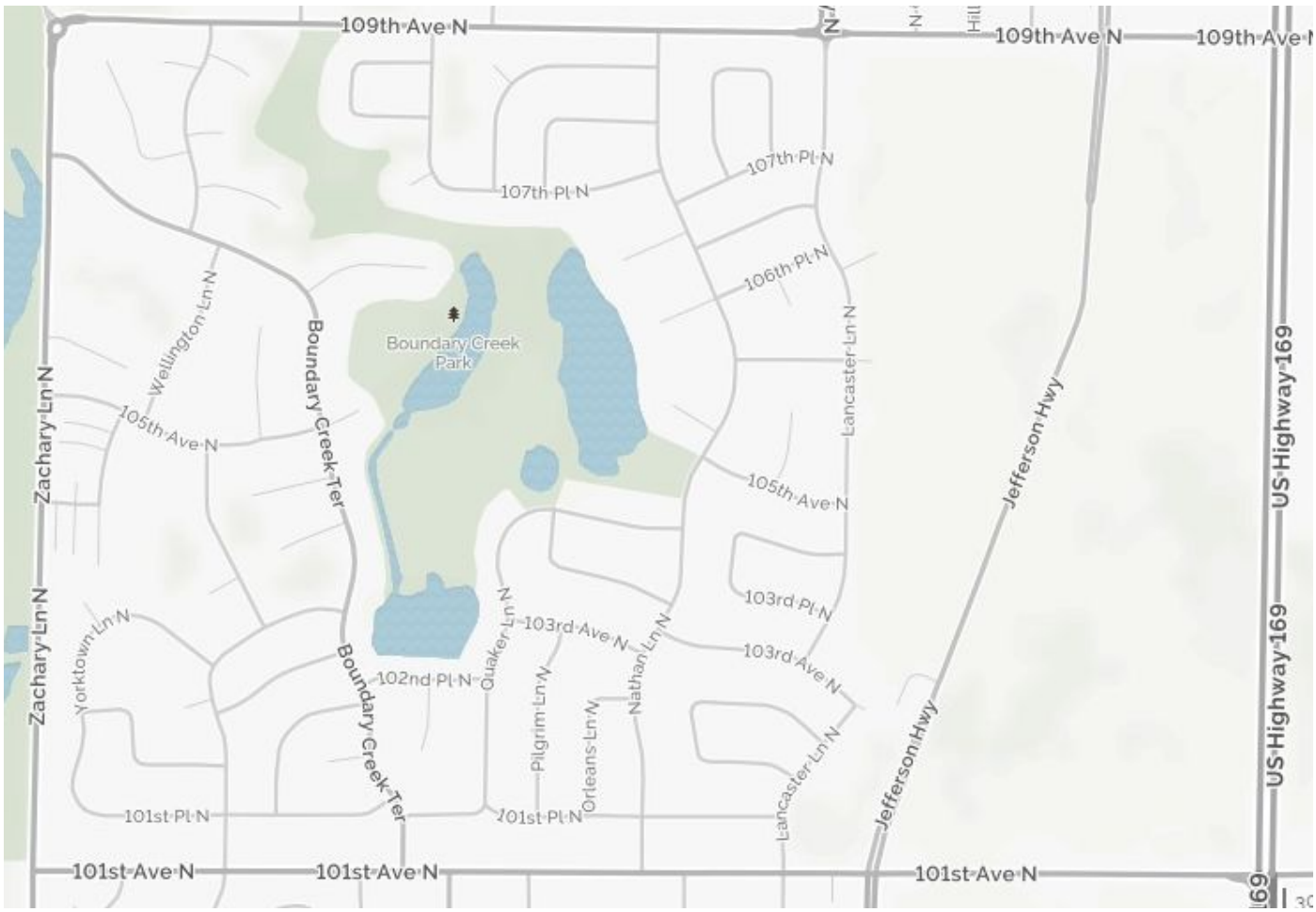
Share: After teachers observes the students working, ask for volunteers to show the class on the board how they created a route and how many times they doubled up on sections of road. What strategies did they explore? Were they effective? Are even and odd degree vertices still a big idea?

Summarize: We saw different methods today of how to approach this problem. We found that the even/odd vertices were not as relevant as they were in Ivanhoe since we had to go 2 directions on each street.

Snow Plowing the Streets of Champlin

Your father is in charge of assigning snow plow operators to plow the city streets of Champlin, MN. The map below is the area (North of and including 101st Ave. N; West of Hwy. 169; South of and including 109th Ave. N.; West of and including Zachary Ln N.) that is giving him some problems. He needs your help to develop the most efficient route for his driver. Some things to note:

1. US Hwy 169 does not need to be plowed by the City of Champlin
2. All streets are bidirectional streets, there are no one way streets in this section of Champlin.
3. Each road has only one lane per direction.



Lesson: Racing Dreams

Racing Dreams from PBS

Benchmark: 7.4.1.1 Design simple experiments and collect data. Determine mean, median and range for quantitative data and from data represented in a display. Use these quantities to draw conclusions about the data, compare different data sets, and make predictions.

Launch: How many of you have ever driven a Go-Kart? Where did you drive it? What was it like? Kart racing is considered the step before going to NASCAR racing for drivers. The NASCAR industry is a multi-million dollar industry each year. For the next couple days we are going to be "racing" on a Kart simulator game called "POV Kart Racing". In all racing, there are several variables that can influence how you could perform in a race. What could some of these variables be in Kart racing? We are going to "play" with some variables and see how they affect the outcomes of our races. The variables we are going to use are: Racing Style, Brand of Engine, and Brand of Tires. You will complete 5 races changing each type of variable and recording your results. Remember to change only 1 variable at a time.

Explore: Students will begin using the "POV Kart Racing" simulator and recording their results on the included chart. Students will also complete the attached worksheet (POV Kart Racer Analysis pdf in link).

Share: When completed students will record on a chart on the smart board which variable most affected their winning. Did the variables have the same affect on everyone's races? Why? Are there any other factors that could affect the results that we did not test today? Looking at your data, which information was the most useful? Was there any data that you really didn't need?

Summarize: We tested different variables and found the mean, median, mode, and range with the data we collected. The mean is the average, the median is the number in the middle, the mode was the most common number, and the range was the distance between our two extremes.

Lesson: M & M Prediction

Benchmark: 7.4.1.1 Design simple experiments and collect data. Determine mean, median and range for quantitative data and from data represented in a display. Use these quantities to draw conclusions about the data, compare different data sets, and make predictions.

Launch: Students will be given a snack size bag of M&M's they will count what is in their bag. Using this information predict what they expect to see in a larger size bag.

Explore: Students will work individually on counting their M&M's and once all student data is collected pool all the data together and have students in groups make a prediction of what is in the larger bag. As students are working questions to ask: Why did you choose those particular numbers? Is there another way that you could've approached it?

Share: Have all the groups share their predictions and the method that they used and why they did that. What is the mean, median, and mode. Which measure of center is better why or why not?

Summarize: Today we made predictions of what is found in an M&M bag. Remember that mean in the average, median is the middle of the road and the mode is the most common.

Lesson: Greatest Yankee Home Run Hitter

Benchmark: 7.4.1.1 Design simple experiments and collect data. Determine mean, median and range for quantitative data and from data represented in a display. Use these quantities to draw conclusions about the data, compare different data sets, and make predictions.

Launch: Your task is to rank the five yankee home run hitters in order from 1 to 5 with 1 being the best home run hitter of all time.You will present your results to the rest of the class. Include graphs and measures of center and spread that reinforce your rankings.

Explore: Students in groups of 2-3 will begin to rank the yankee home run hitters using the statistical method of their choice. As students are working be sure to reiterate to them that they need to be able to justify why they rank a given Yankee player. As students are working suggest to them that they compare your statistical numbers with groups around them because there is a lot of data for some of the players. When students are choosing the types of graphs that they would like to use watch things like labels and scales.

Share: Groups will present their rankings to the class with their reasoning for why they choose which one where and why they choose that particular order. What is something that is important to know about the mean? What is the standard deviation and how did that help some of you make your rankings? In statistics what are some things that are important to keep in mind?

Summarize: The last 2 days we looked at who we believed to be the greatest yankee home run hitter. We looked at using mean, median, mode, standard deviation, and various graphical representations. Mean is the average, median is the middle of the road, and mode is the most common. The mean is affected the most by outliers.

Who was the greatest Yankee Home Run Hitter?

The following table lists five of the greatest New York Yankees' home run hitters with the number of home runs each hit while a Yankee.

Babe Ruth		Lou Gehrig		Mickey Mantle		Roger Maris		Reggie Jackson	
Year	HR	Year	HR	Year	HR	Year	HR	Year	HR
1920	54	1923	1	1951	13	1960	39	1977	32
1921	59	1924	0	1952	23	1961	61	1978	27
1922	35	1925	20	1953	21	1962	33	1979	29
1923	41	1926	16	1954	27	1963	23	1980	41
1924	46	1927	47	1955	37	1964	26	1981	15
1925	25	1928	27	1956	52	1965	8		
1926	47	1929	35	1957	34	1966	13		
1927	60	1930	41	1958	42				
1928	54	1931	46	1959	31				
1929	46	1932	34	1960	40				
1930	49	1933	32	1961	54				
1931	46	1934	49	1962	30				
1932	41	1935	30	1963	15				
1933	34	1936	49	1964	35				
1934	22	1937	37	1965	19				
		1938	29	1966	23				
		1939	0	1967	22				
				1968	18				

Your task is to rank the five players in order from 1 to 5 with 1 being the best home run hitter of all time.You will present your results to the rest of the class. Include graphs and measures of center and spread that reinforce your rankings. Remember…..there is no right or wrong answer here, as long as you can support your decision using math. You can't go just with what you believe you have to support it!

Lesson: Creating a Beautiful Class

Building a Teen Center. Christina, Mary Ann. Key Curriculum Press. Emeryville, CA. !998. P. 24.

Benchmark: 9.3.1.3 Understand that quantities associated with physical measurements must be assigned units; apply such units correctly in expressions, equations and problem solutions that involve measurements; and convert between measurement systems.

9.3.1.5 Make reasonable estimates and judgments about the accuracy of values resulting from calculations involving measurements.

Launch: We are going to make over the classroom! But I need your help to create a proposal for the School Board to approve. Students will be given a ruler and a yardstick.

Explore: Students will work in small groups to explore and develop methods to measure in feet and inches to determine all the dimensions. Discuss in groups how the methods and/or strategies were developed, how and why the methods developed may be different, why and when different methods are used.

Share: Each group will then share and discuss their methods and strategies that were developed. Which part was the easiest? What did you find most difficult to measure? What about the geometric design? Did we all start in the same way? What were the formulas used to determine area?

Summarize: Discuss student's methods and/or strategies, how they developed their methods and what data organization they used. We found the area of the surfaces in the classroom today. We also had to figure out how much paint we will need to paint all the surfaces and a designed geometric figure. We used several different formulas to determine the area, when there are several different methods, mathematicians will choose the most efficient.

Group Activity 2-2

Teen Center Name _____

Creating a Beautiful Classroom

On a separate sheet of paper answer the questions below.

Directions

Have the engineer in your group get a ruler and a yardstick. The recorder will need paper, pencil, and something on which to write.

1. What are the dimensions of the classroom?
2. How high is the ceiling?
3. What are the dimensions of each of the four walls?
4. a. How high is the doorway?
   b. How wide is the doorway?
   c. If you have windows, what are the dimensions of the window(s)?
5. a. If you were going to carpet the classroom, how many square feet of carpet would you need?
   b. How many square yards?
6. If you were going to paint the classroom, what areas would you *not* paint? Describe each area and its dimensions.
7. If you were going to paint the classroom and a gallon of paint covers 400 square feet, how many gallons, quarts, and/or pints would you need to paint the walls, not including the door?
8. Your group has decided to paint the inside door surfaces and the interior trim on the windows a different color. How much paint will you need?
9. Your group decides that after having painted the walls, one wall needs some redecoration. Your group decides to paint some geometric figures including a triangle, hexagon, circle, trapezoid, and rhombus on the wall. Figure out the dimensions and area of each of your figures and then how much paint you would need. (Make a chart.)
10. If the area of your classroom was 120 square feet and its dimensions were 10 by 12 feet, what are other possible dimensions of the room? (Make diagrams.) Which of the possible room dimensions would be the cheapest to carpet? Why?

Lesson: Al Gebra's Night Club

Building a Teen Center. Christina, Mary Ann. Key Curriculum Press. Emeryville, CA. !998. P. 64.

Benchmark: 9.2.2.1 Represent and solve problems in various contexts using linear and quadratic functions.

Launch: Who in here likes to go to dancing? How about who likes to go to parties? Dance parties? Well, Al Gebra is one of the wealthiest men in Litchfield. He has decided to use some of the profits from his other businesses to build a night club at the Teen Center. Just think of the awesome dance parties! After studying much statistical data, Al surmised that this club should seat a minimum of 1152 people at tables. The tables will be 3 feet by 6 feet; and each one will seat 6 customers. Fire codes require that all tables be at least 3 feet from the wall and that there be at least a 3-foot space between tables. There must be a 6-foot walkway through the center of the room parallel to the width of the building.

Al must now decide on the dimensions of the building. Assuming the length of the building will be twice the width, find the dimensions of the smallest possible building Al can construct. Also, describe the arrangement of the tables in the building.

Explore: The students will work in groups of four to complete the following worksheet.

Share: After making observations the teacher should select a few pieces of sample work. Ask those students to present how they arrived at their answers. What were some of the ways that we were able to solve our problem?

Summarize: Today we worked on a layout for a night club. Not every group found the area in the same way. We did find that the most efficient way to answer this project was to use the quadratic equation to find the area we needed for Al to build his night club.

Group Activity 3-3

Teen Center Name _____

Al *Gebra's* Nightclub

Mr. Al Gebra decided to use the profits from another company with which he was very successful to open a teen center. After studying much statistical data, Al surmised that this club should seat a minimum of 1152 people at tables. The tables will be 3 feet by 6 feet, and each one will seat 6 customers. Fire codes require that all tables be at least 3 feet from the wall and that there be at least a 3-foot space between tables. There must be a 6-foot walkway through the center of the room parallel to the width of the building.

Al must now decide on the dimensions of the building. Assuming the length of the building will be twice the width, find the dimensions of the smallest possible building Al can construct. Also, describe the arrangement of the tables in the building.

1. On graph paper, sketch your teen center including the table arrangement. Label all dimensions.
2. Write a summary of your problem-solving procedure. Be specific.
3. List all equations or inequalities used. Be sure to define any variables and/or expressions.